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Testing CPT invariance by using C-even neutral-meson-antimeson correlated states

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Abstract: We consider the decays of a correlated neutral-meson-antimeson state with C-parity +1. We show that there is CPT noninvariance in the mixing of the neutral mesons if, for any two decay modes f and g, the decay rate has a component R_A which is antisymmetric under the interchange of the decay times t_1 and t_2. In particular, one may cleanly extract the CPT-noninvariance parameter with the help of R_A by using opposite-sign dilepton events.

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The system formed by a spin-0 flavoured neutral meson M^0 and its antimeson $\bar{M}^0$ (where M may be either K, D, B_d, or B_s) is experimentally interesting for testing the symmetries CP, T, and CPT [1]. We may recall that CP violation was first observed in the K^0–$\bar{K}^0$ system [2], and is now being actively searched for in the B_d^0–$\bar{B}_d^0$ system [3]; there are also observations of T violation [4] and tests of CPT invariance [5] in the K^0–$\bar{K}^0$ system, although their interpretation remains controversial [6].

We introduce the usual propagation eigenstates

$$|M_H\rangle = p_H|M^0\rangle + q_H|\bar{M}^0\rangle \qquad \text{and} \qquad |M_L\rangle = p_L|M^0\rangle - q_L|\bar{M}^0\rangle\,. \tag{1}$$

These states get multiplied by probability amplitudes $\exp(-i\lambda_H t)$ and $\exp(-i\lambda_L t)$, respectively, for non-zero proper time t. Here,

$$\lambda_H = m_H - \frac{i}{2}\Gamma_H \qquad \text{and} \qquad \lambda_L = m_L - \frac{i}{2}\Gamma_L\,, \tag{2}$$

where m_H, m_L, Γ_H, and Γ_L are real.

Defining [7]

$$\theta = \frac{q_H/p_H - q_L/p_L}{q_H/p_H + q_L/p_L} \qquad \text{and} \qquad \frac{q}{p} = \sqrt{\frac{q_H q_L}{p_H p_L}}\,, \tag{3}$$

one finds that both the real and the imaginary parts of θ are in principle measurable and violate both CP and CPT;[1] while the phase of q/p is convention-dependent and, therefore, unphysical. On the other hand, the magnitude of q/p is measurable and signals T and CP violation in the mixing when it differs from 1.

It is convenient to define the functions

$$g_\pm(t) = \frac{1}{2}\left(e^{-i\lambda_H t} \pm e^{-i\lambda_L t}\right). \tag{4}$$

Then, the probability amplitudes $a(t)$, $b(t)$, $\bar{a}(t)$, and $\bar{b}(t)$ for, respectively, the transitions $M \to M$, $M \to \bar{M}$, $\bar{M} \to \bar{M}$, and $\bar{M} \to M$ are [7, 9]

$$\begin{aligned} a(t) &= g_+(t) - \theta g_-(t)\,, \\ b(t) &= \frac{q}{p}\sqrt{1-\theta^2}\,g_-(t)\,, \\ \bar{a}(t) &= g_+(t) + \theta g_-(t)\,, \\ \bar{b}(t) &= \frac{p}{q}\sqrt{1-\theta^2}\,g_-(t)\,. \end{aligned} \tag{5}$$

[1] For discussions of other CPT-noninvariant observables, see for instance the recent reviews in ref. [8], and the references cited therein.

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In the experimental study of the neutral-meson-antimeson systems it is interesting to use correlated meson-antimeson states of the form

$$|M_\pm\rangle = \frac{1}{\sqrt{2}}\left[|M^0(\vec{k})\rangle \otimes |\bar{M}^0(-\vec{k})\rangle \pm |\bar{M}^0(\vec{k})\rangle \otimes |M^0(-\vec{k})\rangle\right], \tag{6}$$

in which one of the mesons flies in one direction (denoted by the momentum $\vec{k}$) and the other one flies in the opposite direction. Indeed, the DAΦNE experiment at Frascati will use correlated states of the form $|K_-\rangle$, while the Belle and BaBar Collaborations are working with states $|B_{d-}\rangle$. The correlated states $|M_\pm\rangle$ have C-parity ± 1 and are produced from the decay of certain spin-1 resonances like the Φ and $\Upsilon(4S)$. At e^+e^- colliders, states of the type $|M_-\rangle$ are preferred since the produced resonances almost always have quantum numbers $J^{PC} = 1^{--}$. Possible ways of experimentally producing states of the type $|M_+\rangle$ have nevertheless been discussed in the literature, see for instance ref. [10].

It has recently been shown [7] that it is difficult or even impossible to extract θ from experiments with states of the experimentally preferred form $|M_-\rangle$ alone. This happens because, in the semileptonic decays of $|M_-\rangle$, the effects of CPT noninvariance are entangled with those of violations of the $\Delta F = \Delta Q$ rule — where F means flavour, i.e. F may be either S, C, or B. Thus, it is difficult to know whether one is really measuring violations of CPT invariance or one is measuring violations of that rule. For instance, in ref. [11] CPT invariance has been used for checking the rule $\Delta B = \Delta Q$, whereas in ref. [12] θ has been constrained by assuming the validity of this rule.

The purpose of this note is to show that θ can neatly be determined by using the opposite-sign dilepton decays of the symmetrical correlated state $|M_+\rangle$, which has C-parity $+1$. As a matter of fact, tests of CPT invariance using states $|M_+\rangle$ do not necessarily require the use of any specific final states — like the semileptonic decays — and the signal for CPT noninvariance is independent of assumptions about the decay amplitudes to the particular final states that one may wish to use. However, if one wants to extract the actual value of the CPT-violating parameter θ, then it is better to use opposite-sign semileptonic decays.

Let us consider the event in which the meson of $|M_+\rangle$ with momentum $\vec{k}$ decays into f at time t_1, while the meson with momentum $-\vec{k}$ decays into g at time t_2. The rate of such an event is

$$R(f, t_1; g, t_2) = \frac{1}{2}\left|\mathcal{A}(f, t_1; g, t_2)\right|^2, \tag{7}$$

where

$$\begin{aligned}\mathcal{A}(f, t_1; g, t_2) = {} & \left[a(t_1) A_f + b(t_1)\bar{A}_f\right]\left[\bar{b}(t_2) A_g + \bar{a}(t_2)\bar{A}_g\right] + \\ & + \left[\bar{b}(t_1) A_f + \bar{a}(t_1)\bar{A}_f\right]\left[a(t_2) A_g + b(t_2)\bar{A}_g\right],\end{aligned} \tag{8}$$

where A_f is the amplitude for the decay $|M^0\rangle \to |f\rangle$, $\bar{A}_f$ is the amplitude for $|\bar{M}^0\rangle \to |f\rangle$, and similarly for the decays into g. One may use eqs. (5), together with

$$\begin{aligned} g_+(t_1)\, g_+(t_2) + g_-(t_1)\, g_-(t_2) &= g_+(t_1+t_2)\,, \\ g_+(t_1)\, g_-(t_2) + g_-(t_1)\, g_+(t_2) &= g_-(t_1+t_2)\,, \end{aligned} \tag{9}$$

to show that the amplitude $\mathcal{A}$ given in eq. (8) can be written as

$$\begin{aligned} \mathcal{A}(f,t_1;g,t_2) = {} & \frac{p}{q}\sqrt{1-\theta^2}\Big[g_-(t_1+t_2) - 2\theta g_-(t_1)\, g_-(t_2)\Big] A_f A_g + \\ & + \frac{q}{p}\sqrt{1-\theta^2}\Big[g_-(t_1+t_2) + 2\theta g_-(t_1)\, g_-(t_2)\Big] \bar{A}_f \bar{A}_g + \\ & + \Big[g_+(t_1+t_2) - 2\theta^2 g_-(t_1)\, g_-(t_2)\Big] \left(A_f\bar{A}_g + \bar{A}_f A_g\right) + \\ & + \theta\Big[g_+(t_1)\, g_-(t_2) - g_-(t_1)\, g_+(t_2)\Big] \left(A_f\bar{A}_g - \bar{A}_f A_g\right). \end{aligned} \tag{10}$$

Notice that, when $\theta = 0$, the amplitude $\mathcal{A}$ is not only symmetric under the interchange $t_1 \leftrightarrow t_2$ but, as a matter of fact, it is a function only of the sum $t_1 + t_2$ [9, 13]. If, however, the CPT-noninvariance parameter θ is non zero, then $\mathcal{A}$ is not any more a function only of $t_1 + t_2$ and, indeed, it acquires a term — the one in the last line of eq. (10) — *antisymmetric* under $t_1 \leftrightarrow t_2$. Then, the decay rate can be written as the sum of a symmetric component $R_S(f,t_1;g,t_2)$ and an antisymmetric component $R_A(f,t_1;g,t_2)$,

$$\begin{aligned} R_S(f,t_1;g,t_2) &= \frac{1}{2}\Big[R(f,t_1;g,t_2) + R(f,t_2;g,t_1)\Big]\,, \\ R_A(f,t_1;g,t_2) &= \frac{1}{2}\Big[R(f,t_1;g,t_2) - R(f,t_2;g,t_1)\Big]\,, \end{aligned} \tag{11}$$

and *a non-zero R_A arises only if CPT invariance does not hold for neutral-meson mixing* treated in the usual fashion. We conclude that, if experimentally one finds $R_A \neq 0$, then this signals the presence of CPT violation. If, on the other hand, R_A is not significantly different from zero, then this may allow to put a bound on θ.

We want to stress that a measurable CPT-violating asymmetry exists *for any two different final states $f \neq g$* provided $A_f\bar{A}_g - \bar{A}_f A_g \neq 0$; no other assumptions about the amplitudes A_f, A_g, $\bar{A}_f$, and $\bar{A}_g$ are needed — in particular, about their behaviour under the transformations CP, CPT, and T.

As an illustration, let us consider the particular case of opposite-sign dilepton events [14], i.e. inclusive semileptonic decays with $f = X\ell^+\nu_\ell$ and $g = \bar{X}\ell^-\bar{\nu}_\ell$. We shall use the simple notation ℓ^+ for f and ℓ^- for g. We want to show that, in this particular case, it is possible, at least in principle, to explicitly derive the value of θ from the observation of the time dependence of R_A. Allowing for transitions which violate the $\Delta F = \Delta Q$ rule, we introduce the rephasing-invariant quantities [7]

$$\lambda_+ = \frac{q}{p}\frac{\bar{A}_+}{A_+} \qquad \text{and} \qquad \bar{\lambda}_- = \frac{p}{q}\frac{A_-}{\bar{A}_-}\,, \tag{12}$$

where $A_+ \equiv A_{\ell^+}$, $\bar{A}_- \equiv \bar{A}_{\ell^-}$, and so on. As usual, we assume that the quantitities θ, λ_+, and $\bar{\lambda}_-$, which describe "unexpected" physics, are small, and we confine ourselves to the first order in these quantities. If we do this, then the amplitude $\mathcal{A}$ is given by

$$\mathcal{A} = A_+ \bar{A}_- \Big\{ g_+ (t_1 + t_2) + \left(\lambda_+ + \bar{\lambda}_-\right) g_- (t_1 + t_2) + \\ + \theta \Big[g_+ (t_1)\, g_- (t_2) - g_- (t_1)\, g_+ (t_2) \Big] \Big\} . \tag{13}$$

With the usual definitions $\Delta m = m_H - m_L$, $\Delta\Gamma = \Gamma_H - \Gamma_L$, and $\Gamma = (\Gamma_H + \Gamma_L)/2$, we obtain for the decay rate the results

$$R_S \left(\ell^+, t_1; \ell^-, t_2\right) = \frac{1}{8} \left|A_+\right|^2 \left|\bar{A}_-\right|^2 \Big[e^{-\Gamma_H t_+} + e^{-\Gamma_L t_+} + 2\, e^{-\Gamma t_+} \cos\left(\Delta m t_+\right) + \\ + 2 \left(e^{-\Gamma_H t_+} - e^{-\Gamma_L t_+}\right) \mathrm{Re}\, \left(\lambda_+ + \bar{\lambda}_-\right) + \\ + 4\, e^{-\Gamma t_+} \sin\left(\Delta m t_+\right) \mathrm{Im}\, \left(\lambda_+ + \bar{\lambda}_-\right) \Big] \tag{14}$$

and

$$R_A \left(\ell^+, t_1; \ell^-, t_2\right) = \frac{1}{2} \left|A_+\right|^2 \left|\bar{A}_-\right|^2 e^{-\Gamma t_+} \times \\ \times \Bigg\{ \mathrm{Re}\, \theta \left[\sinh\left(\frac{\Delta\Gamma t_1}{2}\right) \cos\left(\Delta m t_2\right) - \\ - \cos\left(\Delta m t_1\right) \sinh\left(\frac{\Delta\Gamma t_2}{2}\right) \right] + \\ + \mathrm{Im}\, \theta \left[\cosh\left(\frac{\Delta\Gamma t_1}{2}\right) \sin\left(\Delta m t_2\right) - \\ - \sin\left(\Delta m t_1\right) \cosh\left(\frac{\Delta\Gamma t_2}{2}\right) \right] \Bigg\} , \tag{15}$$

where $t_+ = t_1 + t_2$. One may note that violations of the $\Delta F = \Delta Q$ rule appear only in R_S, while the CPT-noninvariance parameter θ appears only in R_A, when we take into account "small" physics to first order only.

A glance at eq. (15) confirms that both the real and the imaginary part of θ can be determined using this method. However, in practice that determination depends on the values of Δm and $\Delta\Gamma$. If we use $M = B_d$, we must set $\Delta\Gamma \approx 0$ in eq. (15). Then, only Im θ is measurable and the time dependence is $\exp[-\Gamma(t_1 + t_2)][\sin(\Delta m t_1) - \sin(\Delta m t_2)]$. On the other hand, for $M = K$,[2] where $\Gamma_H \ll \Gamma_L$, we may choose t_1 and t_2 such that $t_1 \sim 1/\Gamma_L \ll t_2$. Equation (15) then yields

$$R_A \left(\ell^+, t_1; \ell^-, t_2\right) \simeq \frac{1}{4} \left|A_+\right|^2 \left|\bar{A}_-\right|^2 e^{-\Gamma t_1 - \Gamma_H t_2} \times \\ \times \Big[\mathrm{Re}\, \theta \cos\left(\Delta m t_1\right) - \mathrm{Im}\, \theta \sin\left(\Delta m t_1\right) \Big] , \tag{16}$$

which is practically independent of t_2 as long as $t_2 \ll 1/\Gamma_H$.

[2]The usual long-lived neutral kaon K_L is our heavier state M_H, and the usual short-lived neutral kaon K_S corresponds to the lighter state M_L.

In conclusion, in this note we have discussed the possibility of using the states $|M_+\rangle$ for probing CPT invariance in the mixing of neutral mesons. We have shown that this is feasible for $A_f \bar{A}_g - \bar{A}_f A_g \neq 0$, because then the decay rate $R(f, t_1; g, t_2)$ has a component which is antisymmetric with respect to the interchange $t_1 \leftrightarrow t_2$ if and only if CPT invariance does not hold. This conclusion is independent of the final states f and g, or any assumptions thereon. We have, in particular, considered the case of opposite-sign dilepton events; then, in the decay rate the effects of CPT violation appear separated from the effects of violation of the $\Delta F = \Delta Q$ rule. It is then possible to cleanly extract the CPT-noninvariance parameter θ, at least when the values of Δm and $\Delta\Gamma$ are not too unfavourable. We believe that, although experimentally it might be hard to work with states $|M_+\rangle$, the cleanness with which they probe CPT invariance might make the effort worthwhile.

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